A Proof of Last Fermat Theorem by Exhaustion

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Abstract. This paper present some results obtained in Excel type software to evaluate some mathematical equations describing the relation between areas of squares that are obtained since Pythagoras Theorem and a useful geometric representation to understand its solutions for triads of natural numbers.

Keyword: Diophantine; Fermat; Pythagoras

[1]

1 Methodology

Lets consider "Golden Triangle" rectangle such as:

Hypotenuse : = c =5

Cathetus adyacent : = b= 4

Cathetus opposite : = a= 3

then for

$a^2 + b^2 = c^2$

we have

$4^2 + 3^2 = 16 + 9 = 5^2 = 25$

Remember that "Pythagoras Theorem" could be represented geometrically in next way

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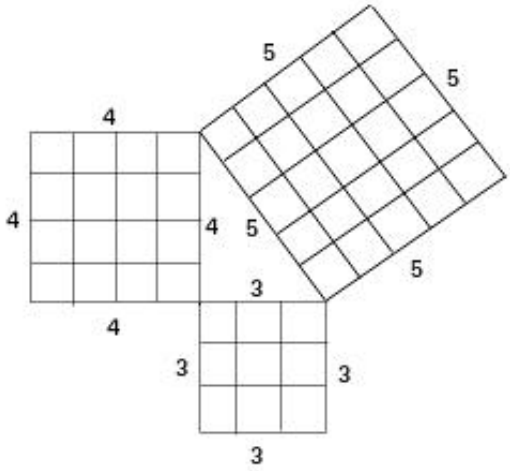

Diagram 1. Graphical representation of Pythagoras Theorem for "Golden Triangle".

Now consider next assumption:

"Areas of the squares of each side of triangle rectangle preserve same percentage proportion for every triad of natural numbers that fulfilled Pythagoras Theorem"

Since Golden Triangle Rectangle we could obtain next relations

If square of side 4 is considered the 100 percentage respect to the square of side 3 then using "Cross Multiplication" we have:

$x_1 = \frac{(9)\cdot(100\%)}{16} = 56.25\%$

For squares of side 5 and 9 we have:

$x_2 = \frac{(9)\cdot(100\%)}{25} = 36\%$

And finally for squares of side 5 and 4:

$x_3 = \frac{(16)\cdot(100\%)}{25} = 64\%$

Generalizing early assumption we have:

$x_1 = \frac{(a^2)\cdot(100\%)}{b^2} = 56.25\%$

$x_2 = \frac{(a^2)\cdot(100\%)}{c^2} = 36\%$

$x_3 = \frac{(b^2)\cdot(100\%)}{c^2} = 64\%$

Clearing a and b from last two equations we have:

$a = c \cdot \sqrt{(\frac{36}{100})}$

$b = c \cdot \sqrt{(\frac{64}{100})}$

Giving values to c; taking only natural number we could obtain next table (values for c are given arbitrarily):

c	a	b	a/b
1	0.6	0.8	0.75
2	1.2	1.6	0.75
3	1.8	2.4	0.75
4	2.4	3.2	0.75
5	3	4	0.75
6	3.6	4.8	0.75
7	4.2	5.6	0.75
8	4.8	6.4	0.75
9	5.4	7.2	0.75
10	6	8	0.75
15	9	12	0.75
20	12	16	0.75
100	60	80	0.75
505	303	404	0.75
1235	741	988	0.75
70000	42000	56000	0.75

Table 1. Values obtained for a and b using constant proportion of areas obtained since Pythagorean Theorem.

We could see that only natural numbers are obtained for the triad a, b, c when c is a multiple of 5. Remember that for x_1 we have $\frac{a}{b} = \sqrt{\frac{56.25}{100}} = 0.75$. Last fact maintains for every $\frac{a}{b}$ either is natural or rational number.

Assuming now that relations for x_1, x_2 and x_3 are given for n-esim power we have:

$x_1 = \frac{(a^n)\cdot(100\%)}{b^n} = 56.25\%$

$x_2 = \frac{(a^n)\cdot(100\%)}{c^n} = 36\%$

$x_3 = \frac{(b^n)\cdot(100\%)}{c^n} = 64\%$

Clearing a and b as before we have:

$a = c \cdot \sqrt[n]{(\frac{36}{100})}$

$b = c \cdot \sqrt[n]{(\frac{64}{100})}$

Taking n = 1 we could obtain next table:

c	a	b	a/b
1	0.36	0.64	0.5625
2	0.72	1.28	0.5625
3	1.08	1.92	0.5625
4	1.44	2.56	0.5625
5	1.8	3.2	0.5625
25	9	16	0.5625
50	18	32	0.5625
100	36	64	0.5625
125	45	80	0.5625
12500	4500	8000	0.5625
125000	45000	80000	0.5625
10000000	3600000	6400000	0.5625
100000000	36000000	64000000	0.5625

Table 2. Values for a and b considering relation defined for x_1, x_2 and x_3.

Last table show proportional relation between areas of squares considered for $n = 1$; where the unique values that fulfilled condition for triad a, b, and c to be natural numbers are those values obtained using main condition when c is a multiple of 25 ; excluding explicitly the rest of natural numbers that could solve the equation:

a + b = c

Same as on table 2 relation $\frac{a}{b} = 0.5625$ maintains.

Taking relation x_1, x_2 and x_3 I present only the case when $c = 5$ and **n** increase to evaluate the numerical relation for higher values of n.

n	c	a	b	a/b	$(a/b)^n$
1	5	1.8	3.2	0.5625	0.5625
2	5	3	4	0.75	0.5625
3	5	3.55689330449006	4.30886938006377	0.825481812223657	0.5625
4	5	3.87298334620742	4.47213595499958	0.866025403784439	0.5625
5	5	4.07596554802961	4.57305051927326	0.891301228983001	0.5625
10	5	4.51440225723717	4.78176249895019	0.944087511294902	0.562499999999999
100	5	4.94917749406452	4.97773536392233	0.994262879046405	0.562499999999999
1000	5	4.9948943523021	4.99776906234324	0.999424801345305	0.562500000000024
10^6	5	4.99999489174637	4.99999776856498	0.999999424636021	0.562500000018183
10^9	5	4.99999999489174	4.99999999776856	0.999999999424636	0.562500030955811
10^{12}	5	4.99999999999489	4.99999999999777	0.999999999999425	0.562526198227803
10^{14}	5	4.99999999999995	4.99999999999998	0.999999999999994	0.555204811793904
10^{15}	5	5	5	1	0.513690767216136

Table 3. Values of a, b for c = 5 and n increasing.

For c = 5; we can see that while n increase **a** and **b** get close to the same value; starting since n=2; when a and b are the values of "Golden Triangle"; this increment is more evident for n = 3. When n = 10; the value of both variables win more accuracy; being this fact showed when both numbers start to have have more number 9 on its significant figures.

Excel and related programs like LibreOffice Calc allows to display of up to 30 decimal places; in the other hand its precision for any specific number is no more than 15 significant figures. Calculations may have an accuracy that is even less due to five issues: round off, truncation, binary storage, accumulation of the deviations of the operands in calculations and cancellation at subtraction of values with similar magnitude.

On table 3, when $n = 10^{15}$; triad shows a = b = c = 5; however the percentage relation condition (a/b) obtained to describe the relation between square areas is different than expected to describe same area´s relation on Pythagoras Theorem. Last fact changes when **n** increase mainly for n = 10 and higher.

In the other hand consider that a/b quotient is of "finite decimal" when n = 1 and n = 2. In any other case such quotient is of "infinite decimal; i.e. a and b are almost equal only when a converge to b and viceversa if c is integer. Using the main condition proposed on this work we can see that a or b are of infinite decimals always c be a natural number and hence a/b will since $n = 3$.

The result of the triad a, b and c leads to the conclusion that "LFT" could have solution for a = b = c = 5, if accuracy of portable electronic devices used to perform calculations is disregarded. Then could be concluded that "LFT" is true for any natural number ($k \in N$; $k \neq 0$) when $a = b = c = k$ for a value

of **n** enough higher to cause the kind of "error" shown by Excel and related programs like LibreOffice Calc.

Besides:

If

$\sqrt[n]{a^n + b^n} = c$

when a = b

$\sqrt[n]{a^n + a^n} = c$

simplifying

$\sqrt[n]{2a^n} = c$

then

$a\sqrt[n]{2} = c$

like a = c then

$a\sqrt[n]{2} = a$

Hence

$\sqrt[n]{2} = \frac{a}{a} = 1$

Therefore

$2 = 1^n$ (false $\forall n \in N$)

Like we´ve "found" computationally that "LFT" can be true for any $a \in N$; establishing triad a = b = c; then we are committed an error in our assumption.

Early results reinforce the idea that "LFT" has solution only for n = 2 for c a multiple of 5, n = 1 for any value of c (or linked with case n = 2, only multiples of 25) and trivial case when $a = 0$, $b = 1$ and $c = 1$ valid for any value of n.

Main "quotient condition" proposed on this work establish a criteria for any value of n on LFT equation $a^n + b^n = c^n$ to to define a, b or c when one of them is given ; having solution on natural numbers only when $c = 1(n \geq 3)$, c multiple of 5 ($n = 2$) or c multiple of 25 (n = 1).

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2 References

This section is empty because until the April of 2024 there is no enough manuscripts available that could be recommended to complete this lecture or that could be useful to prove and understand the content of Last Fermat Theorem.

The only proof that the author of this manuscript could recommend is given by Arne B. Sletsjøe and is presented on the site of Abel Prize like "Popular science articles" with next web direction:

https://abelprize.no/abel-prize-laureates/2016